

Reheating in a brane monodromy inflation model

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ABSTRACT: We study reheating in a recently proposed brane “monodromy inflation” model in which the inflaton is the position of a $D4$ brane on a “twisted torus”. Specifically, we study the repeated collisions between the $D4$ brane and a $D6$ brane (on which the Standard Model fields are assumed to be localized) at a fixed position along the monodromy direction as the $D4$ brane rolls down its potential. We find that there is no trapping of the rolling $D4$ brane until it reaches the bottom of its potential, and that reheating is entirely described by the last brane encounter. Previous collisions have negligible effect on the brane velocity and hence on the reheat temperature. In the context of our setup, reheating is efficient and the reheat temperature is therefore high.

KEYWORDS: Cosmology of Theories beyond the SM, D-branes, Flux compactifications.

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1. Introduction

Among the string theory constructions aiming for plausible early Universe scenarios, brane inflation models are a popular and promising subclass (for reviews see e.g. [1]). They often identify the scalar field ϕ responsible for the early inflationary expansion with the position

of a D -brane of suitable dimensionality [or possibly the distance between several (anti-) D -branes] in the extra, compactified dimensions. This, however, leads to a geometrical upper bound on the field range accessible to such a stringy inflaton: At best, ϕ can travel over the entire extension of the compactified dimension(s), but in most scenarios only part of this range is actually suitable for supporting inflation. Taking into account the inflaton's canonical normalization, its field range was thought to correspond to “small field inflation” (field values smaller than the Planck mass). Since the Lyth bound [2] directly relates the distance travelled in field space to the contribution of tensor modes (gravitational waves) to the observed fluctuations in the cosmic microwave background (CMB) temperature maps, brane inflation scenarios have therefore been known to predict an unobservably low contribution from tensor perturbations ¹.

It turns out that this obstruction to having a large contribution to CMB anisotropies from tensor modes is far from being a no-go theorem. As has recently been emphasized [4], more complicated backgrounds than warped Calabi-Yau manifolds (which abound in brane inflation models) make it possible for the inflaton field to traverse a certain geometric cycle numerous times, adding up to a large distance in field space. The model proposed in [4] is based on a “twisted torus” background for ten-dimensional type IIA superstring theory and is reviewed in the following section of this paper. The inflaton field ϕ corresponds to the position of a $D4$ brane wrapping the twisted torus numerous times.

To obtain a successful inflationary model, it is crucial to consider the exit from inflation and the energy transfer between the inflaton and Standard Model matter fields. In Type IIA theories, Standard Model matter must be localized on branes. In this paper, we study the reheating process assuming that Standard Model matter is confined to a $D6$ brane localized at a certain point along the twisted torus. As the $D4$ brane unwinds, it hits the $D6$ brane numerous times. After the final intersection (after it has unwound completely), the $D4$ brane will come to rest intersecting the $D6$ in $(3+1)$ spacetime dimensions. However, there is the danger that the $D4$ brane might get trapped by the $D6$ at earlier intersections due to strings stretching between them. These strings become massive as the branes separate again, and might thus prevent a graceful exit from inflation. In this paper, we show that this trapping does not occur. Reheating is dominated by the final intersection. We estimate the reheat temperature after inflation and find that it is high.

Reheating in previously proposed brane inflation models has been studied in a large number of papers. Reheating in a brane world model with bulk inflaton was studied in [5] (see also [6]). In a brane-antibrane inflation model the reheating process was investigated by means of the tachyon condensation process in [7] (see also [8] for a study of reheating in a non-inflationary brane-anti brane model). A large body of work is devoted to studying reheating in two throat brane inflation models in which the Standard Model lives in a deeper throat than the one in which the brane-antibrane annihilation process takes place [9, 10]. Reheating through the relaxation of a throat was proposed in [11]. Our mechanism is based on the trapping mechanism by enhanced symmetry states analyzed in [12, 13] (see also [14]

¹As discussed in [3], nonlinear effects due to the primordial scalar metric fluctuations lead to tensor modes, and a lower bound on this contribution can be derived.

for a more recent study). Key to our analysis is the production of open string states in brane interactions, a process studied in detail in [15]. Reheating in an earlier $D6 - D4$ brane inflation model was analyzed in [16]. A study of reheating in a $D3 - D7$ inflation model was presented in [17].

The outline of this paper is as follows: In the following two sections we review the brane inflation model of [4], with particular emphasis on the expressions for the potential energy function in the two limiting regions of field space. In section 4, we derive the relevant equations valid during the slow-roll phase, before going on to our main topic, namely the interaction between the $D4$ and $D6$ branes during the unwinding of the former, and show that (at least for the parameter values preferred in [4]) no trapping occurs. This is done in section 5. In the final section we then estimate the reheat temperature.

2. The model

2.1 The IIA string background

The specific background we consider was constructed in ref. [18] and used for inflationary model building in ref. [4]. This scenario relies on ten-dimensional type IIA string theory with six dimensions compactified on a nilmanifold, that is, a “twisted torus”. This kind of manifold is T-dual to type IIB string theory compactified on a torus with Neveu-Schwarz (NS) flux. Under T-duality, the NS field becomes part of the geometry and leads to a non-trivial fibration of the T-duality cycles over the base. To solve the supergravity equations of motion, this background has to include Ramond-Ramond (RR) two- and four-form flux as well as orientifold planes, but they will not be of concern to us here².

More precisely, the six-dimensional internal manifold is a product of two twisted three-tori. Following the notation of [18], we denote the metric of one of these twisted tori [with coordinates (x, u_1, u_2)] by

$$\frac{ds_{\text{tt}}^2}{\alpha'} = L_{u_1}^2 du_1^2 + L_{u_2}^2 du_2^2 + L_x^2(dx' + Mu_1 du_2)^2, \tag{2.1}$$

where $x' = x - (M/2)u_1u_2$, and M is an integer flux quantum number. At fixed u_1 , the metric of eq. (2.1) describes a torus in the (x', u_2) direction. At $u_1 = 0$, this is simply a square torus, but moving along the u_1 direction, the complex structure τ of this torus changes from $\tau \rightarrow \tau + M$ as $u_1 \rightarrow u_1 + 1$. The manifold is compactified by identifying these two tori, in other words, there is a non-trivial monodromy as we go around the u_1 direction. To be more precise, the manifold is compactified by making the identifications

$$\begin{aligned} (x, u_1, u_2) &\sim (x + 1, u_1, u_2), \\ (x, u_1, u_2) &\sim (x - M/2 u_2, u_1 + 1, u_2), \\ (x, u_1, u_2) &\sim (x + M/2 u_1, u_1, u_2 + 1). \end{aligned} \tag{2.2}$$

²See [18] for details. The setup presented in [4] is laid out in such a way that the orientifold planes do not interfere with the motion of the $D4$ brane.

This means that the coordinates (x, u_1, u_2) are restricted to the interval $[0, 1]$, but with a slight abuse of notation we will let u_1 run over the whole real axis to describe multiple revolutions along this direction.

This background admits a state which gives accelerated expansion because the negative scalar curvature term leads to a positive contribution to the potential energy³. Supersymmetry is broken at a high scale, corresponding to the lowest KK scale of the geometry, because the potential responsible for this is dictated by the curvature of the manifold. In the parameter range of [18], we can assume that the curvature remains weak, but some of the toroidal fibres become very small. We adopt the point of view that all moduli have been stabilized by fluxes as well as the potential due to the non-Kähler structure of the background. This is justified since the inflationary model building in ref. [4] was carried out in the region where these moduli remain fixed and the only dynamical field is the inflaton ϕ , whose interpretation we now discuss.

2.2 Monodromy inflation

To study inflation in this background, one can imagine to wrap a $D4$ brane [with its (4+1)-dimensional worldvolume] along the u_2 direction, its remaining (3+1) dimensions filling the uncompactified dimensions of spacetime. This is not a supersymmetric setup because the background we place the $D4$ in breaks supersymmetry by itself. The $D4$ will aim to minimize its action, and therefore its worldvolume. As mentioned above, the area of the (x', u_2) torus is minimized for $u_1 = 0$, so the $D4$ that wraps a one-cycle inside this torus will unwind by traversing the u_1 cycle several times, until its wrapping number has decreased to 1 and it reaches the position $u_1 = 0$. Using a suitable renormalization derived from the requirement of a canonic kinetic term for the inflaton, the brane position in the u_1 direction is now interpreted as the inflaton field ϕ . Since the nilmanifold has a non-trivial monodromy under $u_1 \rightarrow u_1 + 1$ [see eq. (2.2)], ϕ does not come back onto itself after completing one cycle. It can therefore cross a large field range although the compact coordinate u_1 is restricted to lie in $[0, 1]$ ⁴. In ref. [4] it was shown that inflation in this model takes place for super-Planckian field values with a potential $V(\phi) \propto \phi^{2/3}$, and that $N \sim 60$ e-folds of expansion can be achieved before the slow-roll conditions are violated.

However, to complete this inflationary scenario, one has to specify the mechanism of reheating describing the transition to Standard Model cosmology. To this end, a few more model building ingredients are necessary. In type II theories, the Standard Model is usually constructed on the intersection of stacks of branes on a toroidal orbifold (see e.g. [19] for a review). In type IIA, candidate branes [with at least (3+1) worldvolume dimensions] are $D4$, $D6$ and $D8$ -branes. $D4$ and $D8$ -branes are usually disregarded as they would have to wrap a one- or five-cycle, respectively, both of which are homologically trivial in Calabi-Yau manifolds. The most successful minimal supersymmetric standard models (MSSM) require several stacks of intersecting $D6$ branes, typically using all internal dimensions. We will be less ambitious here and consider only a single $D6$ brane that is wrapped on the second

³For a large enough uplift, [18] also requires supersymmetry breaking KK monopoles (wrapped 5-branes).

⁴See the comment following eq. (2.2) for the use of the range of u_1 .

nilmanifold⁵, which has a metric $d\tilde{s}_{\text{tt}}^2$ analogous to eq. (2.1) with a second set of coordinates $(\tilde{x}', \tilde{u}_1, \tilde{u}_2)$. Again, this is not a supersymmetric setup, but we assume the $D6$ has already minimized its worldvolume, sitting at a meta-stable equilibrium position. We can treat the $D6$ as a probe in the same way we treat the $D4$ as a probe in this background geometry. We choose our setup such that the $D4$ and the $D6$ overlap in three spatial directions but are orthogonal on the internal manifold (this would preserve supersymmetry in flat space). We therefore do not expect any tachyons to appear in the open string spectrum, even after the $D4$ is wrapped on a twisted fibre. An open string stretching between the two branes would only become tachyonic when the branes come very close to each other. At this point, it is the local geometry that matters and the branes are mutually BPS on scales on which the curvature of the background can be neglected. The existence of this tachyon should therefore be insensitive to the global supersymmetry breaking by the background.

During inflation, the $D4$ unwraps and crosses the $D6$, which we choose to localize at $u_1 = 0$, several times. Each time, open string modes between the branes are created. As long as the collision is non-relativistic, only unexcited massless strings are produced [20]. This will indeed turn out to be the case here, the velocity of the inflaton along the slow-roll trajectory being extremely small. These inter-brane strings, however, acquire a mass when the branes move apart from each other again following each collision, and therefore create an attracting force between the $D4$ and the $D6$. This attractive force is in competition with the force (i.e. the inflaton's potential) that tries to unwind the $D4$, aiming to minimize its worldvolume. Note that this additional potential is the main difference to the case studied in ref. [12], where only the string-induced potential was present.

One might worry that this could lead to a trapping of the $D4$ on the $D6$ before a sufficient amount of inflationary expansion has been achieved [12]. It turns out, however, that the number density n_χ of the produced strings at each collision is so small (owing to its dependence $n_\chi \propto |\dot{\phi}|^{3/2}$, with the $D4$'s kinetic energy being extremely small during inflation) that their contribution to the effective potential does not stop the inflaton from rolling down towards the minimum at $u_1 = 0$. Due to the monodromy in the u_1 direction, new strings are produced at each crossing, in principle allowing for an accumulative effect. But as long as each u_1 turn corresponds to several e-folds of slow-roll inflation on the $V(\phi) \propto \phi^{2/3}$ potential, the earlier collisions' string modes have already been diluted to negligible density when the branes meet again. Only towards the end of the unwrapping process does slow-roll break down, and the string densities of the last few collisions may add up. We will discuss this effect in detail below.

Once the $D4$ has unwound completely, it oscillates around the $D6$, reheating it and providing the phenomenological connection to the Universe's subsequent evolution. Simultaneously, when u_1 becomes small, the notion of the renormalized inflaton field ϕ in terms of the coordinate u_1 changes. As a consequence, during this final stage (generically

⁵We could also imagine replacing this nilmanifold by an ordinary toroidal orientifold without twisted fibres. Twisting is only essential on the nilmanifold on which we want to achieve monodromy inflation. The motivation in ref. [4] to have both the nilmanifolds twisted is to make them interchangeable under an orbifold projection and possibly use combinations of their respective coordinates $[(x', u_1, u_2)$ and $(\tilde{x}', \tilde{u}_1, \tilde{u}_2)]$ as candidate inflatons.

after the previous slow-roll phase has ended) the potential is quadratic, $V(\phi) \propto \phi^2$. To estimate the temperature of reheating in this model, we need to determine the velocity of the inflaton when it reaches its potential minimum at $u_1 = 0$ (corresponding to $\phi_0 = 0$) and therefore the minimum of the $D4$ -brane's worldvolume.

3. Inflaton normalization and potential

"Monodromy inflation" is a large field inflation model, i.e. accelerated expansion of the Universe proceeds while the inflaton ϕ moves from a large initial value (measured in Planck units) towards smaller field values. In ref. [4] it was shown that, in order to avoid destabilizing the moduli, the inflaton field has to start below a certain geometry-imposed maximum value ϕ_{\max} . Above this field value the inflaton energy density is so large that the dynamics of the other moduli cannot be neglected. We do not discuss this restriction in detail here and only note that it translates into $\phi_{\max}/M_{\text{Pl}} \simeq \mathcal{O}(10)$ or less, corresponding roughly to the same order of magnitude of turns k_{tot} in the u_1 direction.

The action for the inflaton is derived from the Dirac-Born-Infeld (DBI) world sheet action of the $D4$ brane in the presence of the non-trivial background geometry

$$S_{D4} = \int \frac{d^5\zeta}{(2\pi)^4(\alpha')^{5/2}} e^{-\Phi} \sqrt{\det(G_{MN} + B_{MN})} \partial_\alpha X^M \partial_\beta X^N, \quad (3.1)$$

where $X^M(\zeta_\alpha)$ are the embedding coordinates of the brane, the world sheet coordinates being denoted by ζ_α (Greek indices are world sheet coordinates, capital Latin indices are bulk spacetime coordinates). The bulk metric and bulk NS two-form are G_{MN} and B_{MN} , respectively, so that the argument of the square root gives the pullback of these fields onto the brane worldvolume. The dilaton is denoted by Φ , and the string scale is determined by α' (employing the standard notation from string theory).

Taking the brane to be extended in our three spatial dimensions, uniform in the u_2 direction, and located at the position $u_1(y)$ in the monodromy direction (the coordinates y being our four-dimensional spacetime coordinates), the above action reduces to [4]

$$S_{D4} = \frac{1}{(2\pi)^4 g_s (\alpha')^2} \int d^4 y \sqrt{-g_4} \sqrt{(\beta L_u^2 + L_x^2 M^2(u_1)^2) \left(1 - \alpha' \frac{(L_u)^2}{\beta} \dot{u}_1^2\right)}, \quad (3.2)$$

where $\beta = L_{u_2}/L_{u_1} \equiv L_u^2/(L_{u_1})^2$. L_x, L_{u_1} and L_{u_2} denote the size of the twisted torus in the respective directions, hence β measures the "anisotropy" between the u_1 and u_2 directions and L_u an average over the two. g_s is the string coupling constant whose value is set by the expectation value of the dilaton Φ , g_4 is the determinant of the induced spacetime metric, and the overdot indicates the derivative with respect to physical time.

As is apparent from (3.2), the field $u_1(y)$ is not canonically normalized. For applications to cosmology we need to transform to the corresponding canonically normalized field $\phi(u_1)$, in terms of which the action will then be given by

$$S_{D4} = \int d^4 y \sqrt{-g_4} \left[\frac{1}{2} \dot{\phi}^2 - V(\phi) \right]. \quad (3.3)$$

Expanding the action (3.2) up to two derivatives, we obtain the following form for the potential

$$V(\phi) = \frac{\beta^{1/2} L_u}{(2\pi)^4 g_s \alpha'^2} \sqrt{1 + \frac{M^2 L_x^2}{\beta L_u^2} u_1^2(\phi)}. \quad (3.4)$$

In the small and large field regions, the conversion between the original field u_1 and the canonically normalized field ϕ is of an explicit and simple form. We now list the results (from [4]) and the corresponding potentials.

3.1 At small field values

In the regime $u_1 < u_{1,\text{crit}}$, where $u_{1,\text{crit}}$ (see ref. [4]) is given by

$$u_{1,\text{crit}} \sim \frac{\sqrt{\beta}}{M} \left(\frac{L}{L_x} \right)^{3/2}, \quad L^3 = L_u^2 L_x, \quad (3.5)$$

the potential takes the form

$$V^{\phi < \phi_{\text{crit}}}(\phi) = \frac{m^2}{2} \phi^2, \quad (3.6)$$

where the "mass" m is given in terms of the background parameters by

$$m^2 = \frac{M^2 L_x^4}{\alpha' L^6}. \quad (3.7)$$

In the small field regime, the relation between u_1 and ϕ is linear and given by [4]

$$\frac{\phi}{M_{\text{Pl}}} = \frac{(2\pi)^{3/2} g_s^{1/2} L_u^{3/2}}{\beta^{1/4} L^3} u_1. \quad (3.8)$$

The values of ϕ corresponding to $u_1 < u_{1,\text{crit}}$ are (for the parameter values chosen in [4] which yield a successful inflationary model) much smaller than the four-dimensional Planck mass M_{Pl} , and hence do not lie in the slow-roll regime for inflation.

3.2 At large field values

In the region $u_1 \gg u_{1,\text{crit}}$, the potential takes the form

$$V(\phi) = \mu^{10/3} \phi^{2/3}, \quad (3.9)$$

where the mass scale μ is given in terms of the background parameters by

$$\left(\frac{\mu}{M_{\text{Pl}}} \right)^{10/3} = \left(\frac{3}{2} \right)^{2/3} \frac{1}{(2\pi)^{8/3}} \left(\frac{M^2 \beta}{\alpha'^5 M_{\text{Pl}}^{10} g_s^2} \right)^{1/3} \frac{L_x}{L}. \quad (3.10)$$

If we now calculate the first two slow-roll parameters for the potential of eq. (3.9), we obtain

$$\begin{aligned} \epsilon &\equiv \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_\phi}{V} \right)^2 = \frac{2}{9} \frac{M_{\text{Pl}}^2}{\phi^2}, \\ \eta &\equiv M_{\text{Pl}}^2 \left(\frac{V_{\phi\phi}}{V} \right) = -\epsilon. \end{aligned} \quad (3.11)$$

Hence, it follows that the slow-roll conditions $\epsilon, |\eta| < 1$ both hold until the field reaches the value ϕ_ϵ given by

$$\frac{\phi_\epsilon}{M_{\text{Pl}}} = \sqrt{\frac{2}{9}}. \quad (3.12)$$

For the parameter values used in [4] (see appendix A), the breakdown of the slow-roll approximation occurs in the large field range, i.e.

$$\phi_{\text{crit}} < \phi_\epsilon, \quad (3.13)$$

and we have $\phi > \phi_\epsilon$ for most of the region $\phi \gg \phi_{\text{crit}}$. Hence, slow-roll inflation can occur on the potential (3.9).

For completeness, let us also mention the relation between ϕ and u_1 in the large field regime [4]:

$$\phi = \frac{M^{1/2}}{6\pi^2} \frac{L_u L_x^{1/2}}{(g_s \alpha' \beta)^{1/2}} u_1^{3/2}. \quad (3.14)$$

4. Slow-roll regime

The effective four-dimensional action for the monodromic inflaton ϕ is

$$\mathcal{S} = - \int d^4y \sqrt{-g} \left[\frac{R}{2\kappa} - \frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right], \quad (4.1)$$

where $\kappa = 1/M_{\text{Pl}}^2$, R is the 4d scalar curvature, and the potential $V(\phi)$ is given by eq. (3.9). Assuming the standard spatially flat FLRW metric, the cosmological evolution is then governed by the Friedmann and Klein-Gordon equations,

$$H^2 = \frac{1}{3M_{\text{Pl}}^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) \right], \quad (4.2)$$

$$-V_\phi = \ddot{\phi} + 3H\dot{\phi}. \quad (4.3)$$

Assuming that the slow-roll conditions eqs. (3.11) hold, these equations are well approximated by

$$H^2 \simeq \frac{1}{3M_{\text{Pl}}^2} V(\phi), \quad (4.4)$$

$$-V_\phi \simeq 3H\dot{\phi}. \quad (4.5)$$

It follows immediately that the Hubble parameter scales with μ and ϕ as

$$\frac{H}{M_{\text{Pl}}} \simeq \frac{1}{\sqrt{3}} \left(\frac{\mu}{M_{\text{Pl}}} \right)^{5/3} \left(\frac{\phi}{M_{\text{Pl}}} \right)^{1/3}. \quad (4.6)$$

Likewise, combining eqs. (4.4) and (4.5) gives for the inflaton's velocity

$$\frac{\dot{\phi}}{M_{\text{Pl}}^2} \simeq -\frac{2}{3^{3/2}} \left(\frac{\mu}{M_{\text{Pl}}} \right)^{5/3} \left(\frac{M_{\text{Pl}}}{\phi} \right)^{2/3}. \quad (4.7)$$

The velocity is negative because the field rolls down the potential towards smaller ϕ values (u_1 decreases from its maximum value $u_{1,\text{in}}$ to $u_1 = 0$ as the brane unwraps). The kinetic energy at a given field position then is

$$\frac{E_{\text{kin}}(\phi)}{M_{\text{Pl}}^4} = \frac{2}{3^3} \left(\frac{\mu}{M_{\text{Pl}}} \right)^{10/3} \left(\frac{M_{\text{Pl}}}{\phi} \right)^{4/3}, \quad (4.8)$$

while at the same time, the potential energy amounts to

$$\frac{E_{\text{pot}}(\phi)}{M_{\text{Pl}}^4} = \left(\frac{\mu}{M_{\text{Pl}}} \right)^{10/3} \left(\frac{\phi}{M_{\text{Pl}}} \right)^{2/3}. \quad (4.9)$$

Comparing eqs. (4.9) and (4.8), we see that both energy contributions scale as $\mu^{10/3}$, but $E_{\text{kin}} \propto \phi^{-4/3}$ and $E_{\text{pot}} \propto \phi^{2/3}$. Therefore, roughly speaking, $E_{\text{pot}} > E_{\text{kin}}$ for super-Planckian field values. In particular, note that $E_{\text{pot}} \approx E_{\text{kin}}$ (which occurs when $\epsilon \simeq 1$) around $\phi/M_{\text{Pl}} \simeq \mathcal{O}(1)$, in agreement with the value of ϕ_ϵ from eq. (3.12).

Re-writing eq. (4.5) in terms of the number $N = \int H dt$ of e-foldings of inflation instead of in terms of cosmic time t allows us to integrate and find $N(\phi)$ at a given field value:

$$N(\phi) = \frac{3}{4M_{\text{Pl}}^2} (\phi_{\text{in}}^2 - \phi^2), \quad (4.10)$$

where we assume that inflation starts at the field value ϕ_{in} (thus setting the constant $N_{\text{in}} \equiv N(\phi_{\text{in}}) = 0$). Inverting eq. (4.10), we find for ϕ along the slow-roll trajectory

$$\phi(N) = \sqrt{\phi_{\text{in}}^2 - \frac{4M_{\text{Pl}}^2}{3} N}. \quad (4.11)$$

This equation describes the field evolution until we reach ϕ_ϵ . Provided that $\phi_\epsilon > \phi_{\text{crit}}$, the slow-roll phase is followed by a period when the potential is still given by (3.9), but the motion is too fast to sustain inflation. Then, when the inflaton reaches ϕ_{crit} , the potential becomes quadratic as given by eq. (3.6).

5. Brane collision

We now consider the following situation: The $D4$ brane starts out at some initial field value ϕ_{in} located on the slow-roll trajectory. During its motion down the potential it will collide several times with the $D6$ brane. During the collision process, open strings connecting the two branes will be produced. They will create a restoring force which opposes the further motion of the $D4$ brane. Below we study the strength of this opposing force. We show that, at least for the parameter values assumed in [4], the force is too weak to trap the $D4$ brane.

Let us follow the motion of the $D4$ brane around the torus in the u_1 direction and focus on the collision with the $D6$ which occurs at some position $\phi = \phi_{\text{hit}}$. With the slow-roll trajectory still valid, the impact velocity $\dot{\phi}_{\text{hit}}$ is small, and therefore a non-relativistic treatment of the collision suffices and only unexcited strings are produced [20]. In the string theory picture, at the moment of collision when the branes coincide, open strings are produced with one end attached to each brane. As the $D4$ moves away from the $D6$ again, these strings become massive and try to pull the $D4$ back towards ϕ_{hit} .

5.1 Effective field theory description

At the field theory level, one can model a (non-relativistic) collision [12] by the Lagrangian

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{g_{\phi\chi}^2}{2} (\phi_{\text{hit}} - \phi)^2 \chi^2, \quad (5.1)$$

which describes the coupling of the inflaton ϕ to a field χ that becomes massless at the collision point. In the phenomenological picture at hand, χ stands for the lowest energy modes of the string connecting the two branes.

The Lagrangian (5.1) is of the same type which describes the reheating at the end of inflation in field theory models of inflation. As was discussed in that context [21–24], the equations of motion which follow from (5.1) for the χ field have instabilities which lead to χ particle production with a particle number exponentially increasing in time. Specifically, particle production is concentrated in time intervals during which the evolution of χ is non-adiabatic (see [24] for an in-depth discussion). As shown in [12] and [13] this can lead to the stabilization of moduli fields (like the field ϕ in our case) at enhanced symmetry points. One application of this mechanism is to moduli stabilization in string gas cosmology [25, 26] (see [27] for a discussion of this application).

Returning to our discussion, the mass of the χ particles increases in proportion to the distance between the two branes, i.e.

$$m_\chi^2(t) = g_{\phi\chi}^2 (\phi_{\text{hit}} - \phi)^2, \quad (5.2)$$

where $g_{\phi\chi}^2$ is related to the string coupling constant. Thus, the time-dependent frequency for the k th mode of the χ field is given by

$$\omega(t) = \sqrt{k^2 + m_\chi^2(t)}, \quad (5.3)$$

and one can calculate the “adiabaticity parameter”

$$\left| \frac{\dot{\omega}}{\omega^2} \right| \approx \frac{|\dot{\phi}|}{g_{\phi\chi}} \frac{1}{(\phi_{\text{hit}} - \phi)^2}. \quad (5.4)$$

This is greater than $\mathcal{O}(1)$ in the interval

$$|\Delta\phi| \leq \sqrt{\frac{|\dot{\phi}|}{g_{\phi\chi}}}, \quad (5.5)$$

where $\dot{\phi}$ is evaluated at the collision time. Taking this velocity from eq. (4.7) (valid while ϕ is on the slow-roll trajectory) gives

$$\left| \frac{\Delta\phi}{M_{\text{Pl}}} \right| \leq \frac{\sqrt{2}}{3^{3/4} g_{\phi\chi}^{1/2}} \left(\frac{\mu}{M_{\text{Pl}}} \right)^{5/6} \left(\frac{M_{\text{Pl}}}{\phi} \right)^{1/3}. \quad (5.6)$$

For the parameter values of [4], the value of this expression is much smaller than 1. It is within this small field range that particle/string creation occurs. In particular, we expect

this process to be instantaneous compared to the Hubble time H^{-1} . At the speed (4.7), the field needs a time interval

$$\Delta t H = \frac{3^{1/4}}{\sqrt{2g_{\phi\chi}}} \left(\frac{\mu}{M_{\text{Pl}}}\right)^{5/6} \left(\frac{\phi}{M_{\text{Pl}}}\right)^{2/3} \quad (5.7)$$

[where we have made use of eq. (4.6) to determine the Hubble parameter] to pass through the field range (5.5). This is small compared to one (and hence the time Δt short compared to the Hubble scale) if the field value ϕ satisfies

$$\frac{\phi}{M_{\text{Pl}}} \ll \left(\frac{M_{\text{Pl}}}{\mu}\right)^{5/4} \left(\frac{2g_{\phi\chi}}{\sqrt{3}}\right)^{3/4}. \quad (5.8)$$

For the parameter values discussed in appendix A this is always the case for ϕ values sufficiently small such that moduli stabilization is not jeopardized (see ref. [4]). Therefore it is justified to treat the process of string creation as instantaneous while the inflaton is on its slow-roll trajectory.

The force on the $D4$ brane created by the strings yields a contribution to the effective potential which determines the motion of the $D4$ brane after its encounter with the $D6$ brane. Note, however, that unlike the example of ref. [12], our Lagrangian (5.1) already contains a bare potential for ϕ [given by eq. (3.9)] before the collision. The total effective potential $V_{\text{eff}}(\phi)$ after the brane encounter will therefore comprise the old $V(\phi)$ as well as the induced string contribution.

According to ref. [12], the number density of produced χ particles in a head-on brane collision is given by

$$n_{\chi} = \frac{\left(g_{\phi\chi}|\dot{\phi}_{\text{hit}}|\right)^{3/2}}{(2\pi)^3}, \quad (5.9)$$

where $\dot{\phi}_{\text{hit}}$ is the field velocity upon impact. As the $D4$ moves past the $D6$, these created particles produce a contribution to the potential for the ϕ field of the form

$$\rho_{\chi}(\phi) = g_{\phi\chi} n_{\chi} (\phi_{\text{hit}} - \phi). \quad (5.10)$$

Assuming that we are on the slow-roll trajectory and that therefore (4.7) can be used, this potential becomes

$$\rho_{\chi}(\phi) = \frac{g_{\phi\chi}^{5/2} M_{\text{Pl}}^4}{2^{3/2} \pi^3 3^{9/4}} \left(\frac{\mu}{M_{\text{Pl}}}\right)^{5/2} \left(\frac{M_{\text{Pl}}}{\phi_{\text{hit}}}\right) \left(\frac{\phi_{\text{hit}} - \phi}{M_{\text{Pl}}}\right). \quad (5.11)$$

It is possible that this new potential energy contribution creates a local minimum in which the inflaton might get trapped. Our next goal is to evaluate whether the $D4$ comes to a halt after its encounter with the $D6$ brane located at ϕ_{hit} .

5.2 Trapping effects

Before the $D4$ hits the $D6$, ϕ is moving along the slow-roll trajectory (4.11) that was obtained from eqs. (4.4)–(4.5) with the potential (3.9). In this way, we can calculate the

impact velocity $\dot{\phi}_{\text{hit}}$ and therefore the string production rate at the time of the collision:

$$\frac{n_{\chi}^{(1)}}{M_{\text{Pl}}^3} = \frac{2^{3/2} g_{\phi\chi}^{3/2}}{(2\pi)^3 3^{9/4}} \left(\frac{\mu}{M_{\text{Pl}}} \right)^{5/2} \frac{M_{\text{Pl}}}{\dot{\phi}_{\text{hit}}}. \quad (5.12)$$

After the brane collision, the effective potential consists of eq. (3.9) plus the contribution from the newly created strings:

$$V_{\text{eff}}(\phi) = \mu^{10/3} \phi^{2/3} + g_{\phi\chi} n_{\chi}^{(1)} (\phi_{\text{hit}} - \phi). \quad (5.13)$$

Let us now check whether the field can come to a rest before it rounds the torus and hits the $D6$ brane a second time (i.e. between the values of $u_{1,\text{hit}}$ and $u_{1,\text{hit}} - 1$). A necessary condition for the potential (5.13) to develop a local minimum is for V'_{eff} to change sign. As the bare potential (3.9) has a monotonic $V' > 0$, we have to check whether $V'_{\text{eff}} < 0$ is possible. This amounts to requiring

$$g_{\phi\chi} n_{\chi} > \frac{2}{3} \frac{\mu^{10/3}}{\phi^{1/3}}, \quad (5.14)$$

which translates into

$$\frac{\phi}{M_{\text{Pl}}} > \left[\frac{(2\pi)^3 3^{5/4}}{2^{1/2} g_{\phi\chi}^{5/2}} \right]^3 \left(\frac{\mu}{M_{\text{Pl}}} \right)^{5/2} \left(\frac{\phi_{\text{hit}}}{M_{\text{Pl}}} \right)^3. \quad (5.15)$$

For the parameter values of [4] the required field values are much larger than those in the slow-roll region. Hence we conclude that the strings produced in a single encounter are too weak to trap the inflaton field.

One may now worry that the buildup of strings produced in various encounters might trap the inflaton. A second encounter will generate new strings between the branes, while those from the first collision become heavier and heavier.⁶ The number density n_{χ} of strings is larger at later encounters because the velocity of the inflaton at the impact point increases [see eq. (4.7)]. However, as long as the field remains in the slow-roll region, the increase in velocity is small. At the same time, however, space is inflating and thus the number density of the strings is decreasing exponentially. Thus, strings produced at previous encounters have a negligible effect on the later encounters as long as the time interval corresponding to successive encounters is longer than a Hubble expansion time. The redshifting of the number density of strings also ensures that the correction to the effective potential has a negligible effect on the evolution of the inflaton.

We have seen that the strings created after a single collision have a negligible effect on the field trajectory. Thus, it is reasonable to assume that even after the first brane encounter at ϕ_{hit} the slow-roll trajectory remains valid. For simplicity, let us use eq. (4.7) all the way while $\phi > \phi_{\epsilon}$.⁷ It is then easy to check for the parameters of [4] that no trapping

⁶We ignore the effect of strings reattaching to the $D6$ brane at the second collision, which would reduce the attractive force between the branes.

⁷Note that, while eq. (4.7) should be modified after each brane collision because of the additional potential terms created by each new generation of inter-brane strings, these strings will, if anything, *slow down* the brane motion further. Hence, the field in reality would be rolling even slower than (4.7) tells us. Since the string production rate $n_{\chi} \propto |\dot{\phi}|^{3/2}$, we are therefore *overestimating* the effect of string production.

can occur while the inflaton is on the slow-roll trajectory. Correspondingly, eq. (4.10) tells us how many e-folds are produced at each turn. At the beginning of inflation, this number is $\Delta N \simeq \mathcal{O}(10)$. It is evident that this expansion of space dilutes all the strings created at the first hit to the extent that they do not play a role at the second encounter. However, once the expansion drops to $\Delta N < 1$ per turn, the created strings may indeed accumulate. The crucial question is whether the potential contribution (5.10) induced by them can bring the motion of the $D4$ to a stop before it reaches the minimum of the original potential eq. (3.9) at $u_1 = 0$ (corresponding to $\phi_0 = 0$). If so, one may wonder how much energy leaks from ϕ into the fields localized on the $D6$ brane (hence reheating them) while inflation is still under way. We will show that even with an overestimation of the string effect its influence is negligible.

6. Reheat temperature

In this section, we first estimate the reheat temperature at the final brane collision ($u_1 = 0$) ignoring the effect of string creation at previous brane encounters, and then refine this calculation taking into account the strings created during the last few turns, which do not get diluted any more by the inflationary expansion.

6.1 Neglecting string production

The slow-roll trajectory is valid up to ϕ_ϵ , at which point the kinetic and potential energies become equal. Therefore, we can estimate the total energy at ϕ_ϵ as

$$\frac{E_{\text{tot}}(\phi_\epsilon)}{M_{\text{Pl}}^4} \simeq 2 \left(\frac{\mu}{M_{\text{Pl}}} \right)^{10/3} \left(\frac{\phi_\epsilon}{M_{\text{Pl}}} \right)^{2/3}. \quad (6.1)$$

Since after the breakdown of slow-roll the amount of energy which is lost to the expansion of space is negligible [assuming that the reheating process is rapid, an assumption whose validity is assured by the estimate (5.7)], the total energy (6.1) is approximately conserved down to $u_1 = 0$. At that point, no inflaton potential energy is left [the $D4$ having minimized its worldvolume, see eq. (3.9)], and hence the entire E_{tot} of eq. (6.1) is converted into kinetic energy. Therefore, the velocity of the brane when it reaches $u_1 = 0$ (corresponding to $\phi_0 = 0$) is given by

$$\frac{|\dot{\phi}_0|}{M_{\text{Pl}}^2} = \sqrt{\frac{2E_{\text{tot}}(\phi_\epsilon)}{M_{\text{Pl}}^4}} = 2 \left(\frac{\mu}{M_{\text{Pl}}} \right)^{5/3} \left(\frac{\phi_\epsilon}{M_{\text{Pl}}} \right)^{1/3}. \quad (6.2)$$

6.1.1 Reheat temperature from single impact

To calculate the reheat temperature, let us first determine how much energy is channelled from the ϕ to the χ field at the final encounter itself. To this end, we set [compare eq. (5.9)]

$$n_\chi^{(u_1=0)} = (T_{\text{rh}}^\chi)^3 = \frac{g_{\phi\chi}^{5/2} |\dot{\phi}_0|^{3/2}}{(2\pi)^3}, \quad (6.3)$$

assuming that the energy released as χ particles rapidly thermalizes (otherwise it would not make sense to talk about a temperature). Given this assumption, the reheat temperature is

$$T_{\text{rh}}^{\chi} = \frac{g_{\phi\chi}^{5/6}}{2\pi} |\dot{\phi}_0|^{1/2} = \frac{g_{\phi\chi}^{5/6}}{2^{3/4}\pi} [E_{\text{tot}}(\phi_{\epsilon})]^{1/4}. \quad (6.4)$$

With the estimate (6.1), this becomes

$$\frac{T_{\text{rh}}^{\chi}}{M_{\text{Pl}}} = \frac{g_{\phi\chi}^{5/6}}{\sqrt{2}\pi} \left(\frac{\mu}{M_{\text{Pl}}}\right)^{5/6} \left(\frac{\phi_{\epsilon}}{M_{\text{Pl}}}\right)^{1/6}. \quad (6.5)$$

Note, however, that this only accounts for the energy transferred into the χ field on the first hit after the unwrapping process comes to an end. During reheating, the $D4$ oscillates around the $D6$, gradually channelling more energy into the χ field. We can estimate the amplitude of these oscillations from (see [12])

$$\phi_{\text{osc}} = \frac{4\pi^3}{g_{\phi\chi}^{5/2}} |\dot{\phi}_0|^{1/2}. \quad (6.6)$$

We can compare this again to the region in which the particle production is effective (compare section 5.1), leading to

$$\frac{\Delta\phi}{\phi_{\text{osc}}} = \frac{g_{\phi\chi}^2}{4\pi^3}. \quad (6.7)$$

For a perturbative value of the coupling, the particle production therefore still occurs only during a small fraction of an oscillation.

6.1.2 Reheat temperature from entire energy transfer

If the $D4$ brane comes eventually to a stop, all of its energy (apart from its rest mass, which we have consistently ignored in the whole analysis) will go into particles (modulo energy which is lost into closed string modes, e.g. bulk gravitons⁸). Thus, we can estimate the final reheat temperature by simply equating the final thermal energy with the inflaton energy at the beginning of the reheating phase, i.e. using

$$\rho_{\text{rh}} = \frac{\pi^2}{30} g_* \left(T_{\text{rh}}^{\text{rad}}\right)^4, \quad (6.8)$$

where ρ_{rh} is the energy density at the beginning of the reheating phase. Here, g_* is the number of spin degrees of freedom in the final bath of radiative particles. Taking the number from Standard Model particle physics, it is a constant of $\mathcal{O}(10^2)$. With eq. (6.1), this gives a reheat temperature of

$$T_{\text{rh}}^{\text{rad}} = \left(\frac{60}{g_*\pi^2}\right)^{1/4} \left(\frac{\mu}{M_{\text{Pl}}}\right)^{5/6} \left(\frac{\phi_{\epsilon}}{M_{\text{Pl}}}\right)^{1/6}. \quad (6.9)$$

⁸However, for the parameters we are using for which the Planck length is smaller than the string length, the decay into bulk particles will be suppressed.

We immediately see the same functional dependence on μ and ϕ_ϵ as in eq. (6.5). Comparing eq. (6.9) with this previous result, we see that in order to achieve the same total energy transfer, the number of oscillations n_{osc} of the $D4$ through the $D6$ can be estimated as

$$n_{\text{osc}} = \left(\frac{60\pi^2}{g_*} \right)^{1/4} \frac{\sqrt{2}}{g_{\phi\chi}^{5/6}}. \quad (6.10)$$

6.2 Cumulative effect of wound strings

We now refine the above calculation by including the string-induced corrections. Our argument shall be directed towards finding a lower bound on the reheat temperature. We will therefore *overestimate* the contribution of the strings that are created during brane collision. As already argued earlier, as long as each turn corresponds to several e-folds, we can safely assume that all open string states are diluted to a negligible extent. However, as soon as this is not true anymore, some fraction of the strings will survive and get wound around the torus multiple times. (This is the case if we neglect the possibility that they reattach to the $D6$ brane. In principle a 4-6 string can split into a 6-6 string — that would wind exactly once — and a massless 6-4 string at the time of the next crossing between the $D4$ and $D6$.) In consistency with overestimating the string effect, we will assume that the point from which on strings do not get diluted completely anymore occurs at some value of $u_1 = k$, which is higher than where slow roll breaks down at $u_1 = k_{\text{sr}}$. Furthermore, we will assume that not only a fraction of them survives, but all of them.

Then our strategy to determine the final velocity $\dot{\phi}_0$ at $u_1 = 0$ is the following: We use a slow roll trajectory all the way down to $u_1 = k_{\text{sr}}$ from which point on we assume that no energy is lost to the expansion of space anymore. We do, however, include the strings created between the turns $u_1 = k$ and the end of slow-roll in the potential

$$V_{\text{eff}}(\phi_{\text{sr}}) = \mu^{10/3} \phi_{\text{sr}}^{2/3} + \frac{g_{\phi\chi}^{5/2}}{(2\pi)^3} \sum_{i=k_{\text{sr}}+1}^k v^{3/2}(u_1 = i) [\phi(u_1 = i) - \phi_{\text{sr}}]. \quad (6.11)$$

Note that here we calculate the inflaton velocity $v(u_1 = i)$ from the original trajectory (4.7), which is consistent with our overestimate: a higher velocity corresponds to a higher string production rate. Making use of (6.11) we obtain the inflaton velocity at the end of slow-roll via

$$\dot{\phi}_{\text{sr}} = \frac{V'_{\text{eff}}}{\sqrt{3V_{\text{eff}}}} M_{\text{Pl}}. \quad (6.12)$$

This enters into the total energy at the point where slow-roll ends

$$E_{\text{tot}}(u_1 = k_{\text{sr}}) = \frac{1}{2} \dot{\phi}_{\text{sr}}^2 + \mu^{10/3} \phi_{\text{sr}}^{2/3} + \frac{g_{\phi\chi}^{5/2}}{(2\pi)^3} \sum_{i=k_{\text{sr}}+1}^k v^{3/2}(u_1 = i) [\phi(u_1 = i) - \phi_{\text{sr}}], \quad (6.13)$$

which we assume to remain conserved from now on. It will mostly be converted into kinetic energy, as the original potential vanishes at $\phi = 0$, so the total final energy reads

$$E_{\text{tot}}(\phi_0 = 0) = \frac{1}{2} \dot{\phi}_0^2 + \frac{g_{\phi\chi}^{5/2}}{(2\pi)^3} \sum_{i=k_{\text{sr}}+1}^k v^{3/2}(u_1 = i) \phi(u_1 = i) + \frac{g_{\phi\chi}^{5/2}}{(2\pi)^3} \sum_{i=1}^{k_{\text{sr}}} \tilde{v}^{3/2}(u_1 = i) \phi(u_1 = i), \quad (6.14)$$

where the last line denotes the contribution from additional strings created during the last few revolutions when slow-roll has ended. These last terms are a bit more complicated to calculate, as we now have to determine the velocity \tilde{v} for $u_1 < k_{\text{sr}}$ from energy conservation instead of from the slow-roll trajectory. This is further complicated by the fact that below ϕ_{crit} the potential changes to $m^2\phi^2$, see eq. (3.6). However, in the numerical example we study below, it turns out that ϕ_{crit} corresponds to less than one revolution in the u_1 direction, so we do not have to worry about this fact at all. Also, the iteration necessary to determine the velocities for $u_1 < k_{\text{sr}}$ is not as messy as it seems, since in our example $k_{\text{sr}} = 2$.

Equating the energies (6.13) and (6.14) we finally arrive at the kinetic energy of the inflaton when it reaches the minimum

$$E_{\text{kin}}(u_1 = 0) = \frac{1}{2} \dot{\phi}_{\text{sr}}^2 + \mu^{10/3} \phi_{\text{sr}}^{2/3} - \frac{g_{\phi\chi}^{5/2}}{(2\pi)^3} \sum_{i=k_{\text{sr}}+1}^k v^{3/2}(u_1 = i) \phi_{\text{sr}} \quad (6.15)$$

$$- \frac{g_{\phi\chi}^{5/2}}{(2\pi)^3} \sum_{i=1}^{k_{\text{sr}}} \tilde{v}^{3/2}(u_1 = i) \phi(u_1 = i).$$

With this expression we can now use eqs. (6.3) or (6.8) to infer the reheat temperature found from our refined calculation. Since we have taken into account additional energy loss, the result in each case should be a smaller T_{rh} than those of section 6.1. We find

$$T_{\text{rh}}^{\chi} = \frac{g_{\phi\chi}^{5/6}}{2^{3/4}\pi} [E_{\text{kin}}(\phi_0)]^{1/4} \quad \text{and} \quad (6.16)$$

$$T_{\text{rh}}^{\text{rad}} = \left[\frac{30g_*}{\pi^2} E_{\text{kin}}(\phi_0) \right]^{1/4}, \quad (6.17)$$

from the reasoning following a coupling to the χ field (section 6.1.1) or energy transfer into radiation (section 6.1.2), respectively.

In its full generality, the calculation presented in this subsection seems rather involved. However, in a concrete numerical example, the reasoning is much more intuitive since the total number k of turns that qualify for undiluted string production is small. We now turn to studying such an example to illustrate the effect of string production on the reheat temperature in the monodromy inflation model at hand.

7. Numerical example

We illustrate the relations derived above with a numerical example using the parameter values employed in ref. [4]. For convenience, we summarily list these parameter values along with some useful relations in appendix A. Inserting these values, the critical u_1 and ϕ values from eqs. (3.5) and (3.8) become

$$u_{1,\text{crit}} \simeq 0.7, \quad \phi_{\text{crit}} = 0.1M_{\text{Pl}}. \quad (7.1)$$

Therefore, the large field approximation only breaks down during the last u_1 turn, and we can safely use the corresponding potential eq. (3.9) up to that point. From eq. (3.12) it follows that no slow-roll is possible in the region $\phi < \phi_{\text{crit}}$ with eq. (7.1).

We set the initial u_1 value to be the largest value for which it makes sense to focus on the dynamics of the candidate inflaton field alone (see the corresponding discussion in the first paragraph of section 3). According to the analysis in [4], this correspond to a field value of about $10M_{\text{Pl}}$. Specifically, we take the first brane collision to occur at

$$u_{1,\text{hit}} = 13, \quad \text{corresponding to} \quad \phi_{\text{hit}} \simeq 9.1M_{\text{Pl}}. \quad (7.2)$$

That is, the $D4$ brane is initially wrapped slightly more than thirteen times along the u_1 direction to ensure $\phi_{\text{in}} > \phi_{\text{hit}}$.

The breakdown of slow-roll occurs [see eq. (3.12)] at the field value

$$\phi_\epsilon \simeq 0.5M_{\text{Pl}}, \quad u_{1,\epsilon} \simeq 1.8, \quad (7.3)$$

and therefore only the last two u_1 turns do not occur entirely in the slow-roll regime.

For the scale μ of the potential, we find from eq. (3.10) that

$$\left(\frac{\mu}{M_{\text{Pl}}}\right)^{10/3} \simeq 8.7 \cdot 10^{-10}. \quad (7.4)$$

This gives for the initial Hubble scale and velocity

$$H_{\text{in}} \simeq 3.6 \cdot 10^{-5}M_{\text{Pl}}, \quad \dot{\phi}_{\text{in}} \simeq -2.6 \cdot 10^{-6}M_{\text{Pl}}^2, \quad (7.5)$$

illustrating that our non-relativistic treatment of the brane collisions is well justified.

Turning to the effective description of the brane collisions, we have to specify the coupling between the fields $g_{\phi\chi}$ in addition to the parameters of appendix A. We take

$$g_{\phi\chi} \simeq 0.1, \quad (7.6)$$

unless otherwise stated.⁹ Then, for the first collision occurring at ϕ_{hit} , the interaction takes place over a range [see eq. (5.6)]

$$\frac{\Delta\phi}{M_{\text{Pl}}} \simeq 5.1 \cdot 10^{-3}, \quad (7.7)$$

and from eq. (5.7) one obtains that the interaction time is $\mathcal{O}(10^{-2})$ smaller than the Hubble time, i.e. quasi-instantaneous.

With respect to the question whether the inflaton can get trapped, we keep $g_{\phi\chi}$ unfixed for the moment. We argued earlier that eq. (5.15) remains valid along the whole slow roll trajectory, which means down to $u_1 = 2$ in our case. Inserting our numerical values into eq. (5.15) we find that trapping can occur only if

$$\frac{\phi}{M_{\text{Pl}}} > \frac{0.3}{g_{\phi\chi}^{15/2}}. \quad (7.8)$$

⁹This does not mean we are equating the string coupling to the coupling that describes the creation of strings on the effective field theory level in general. We just want to assume a reasonable perturbative value for $g_{\phi\chi}$.

For our choice of the coupling constant $g_{\phi\chi} = 0.1$, this corresponds to a field value much larger than the maximum field value ϕ_{\max} (see the beginning of section 3). If we were to impose that the field should stop after the first collision of the two branes we would require a coupling between ϕ and χ of $g_{\phi\chi} \approx 3$. Even though the string production rate increases towards smaller ϕ , for the smallest value that is still on the slow roll trajectory (corresponding to $u_1 = 2$) one would require $g_{\phi\chi} \approx 1.4$ for trapping to occur. Hence for a perturbative field coupling the $D4$ cannot become trapped. One could repeat this estimate for the last two turns, which lie outside the slow roll regime, by assuming that no energy is lost to the expansion of space anymore and that the heavy strings accumulate. However, as we will see shortly, even if we overestimate the string effect its influence remains negligible.

7.1 Reheating neglecting string production

Let us first estimate the reheat temperature when the string production effects are ignored. Then, considering reheating through χ from eq. (6.5) with values of ϕ_ϵ and μ corresponding to our parameters, we find

$$T_{\text{rh}}^\chi \simeq 1.6 \cdot 10^{-4} M_{\text{Pl}} \approx 3.9 \cdot 10^{14} \text{ GeV}, \quad (7.9)$$

which is a very high reheat temperature. If we consider the overall energy transfer into radiation, the reheat temperature found from eq. (6.9) is (using $g_* = 100$)

$$T_{\text{rh}}^{\text{rad}} \simeq 2.4 \cdot 10^{-3} M_{\text{Pl}} \approx 5.8 \cdot 10^{15} \text{ GeV}, \quad (7.10)$$

which would take about [see eq. (6.10)]

$$n_{\text{osc}} \simeq 15 \quad (7.11)$$

oscillations around the $D6$ located at $u_1 = 0$.

7.2 Reheating with strings from brane collisions

Clearly, the previous result is an overestimate of the reheat temperature since it ignores the energy loss due to string creation and stretching between the branes. Another way to see this is as follows: due to the production of strings, the $D4$ brane will be moving slightly slower towards the end of the inflationary phase. The slow-rolling approximation will be valid until a smaller value of the field, and thus at the time of exit from slow-rolling the $D4$ brane will carry less energy. Let us therefore now turn to the more refined calculation of section 6.2 in our concrete numerical example.

From eq. (4.10) we can calculate how many e-folds of inflation are produced at each u_1 turn in the slow-roll regime. For example, on the first turn between $u_{1,\text{hit}} = 13$ and $u_1 = 12$, $\Delta N_{13 \rightarrow 12} \simeq 13$ e-folds are produced. However, between $u_1 = 5$ and $u_1 = 4$, this has dropped to $\Delta N_{5 \rightarrow 4} \approx 2$, and in the following turn only one e-fold of expansion is produced, $\Delta N_{4 \rightarrow 3} \approx 1$. Hence, when looking for a careful reheat temperature estimate, we cannot assume that the strings created at and after $u_1 = 5$ are diluted to a negligible density. Therefore we set $k = 5$ in the notation of section 6.2. We also know, see eq. (7.3),

that $k_{\text{sr}} = 2$ since $u_1 = 2$ is the last turn to occur in the slow-roll regime, and it corresponds to $\phi_{\text{sr}} \simeq 0.6M_{\text{Pl}}$. If we work through the numerics, always considering that we are still in the slow-roll regime when calculating velocities, we find for the total inflaton energy (6.13) that will remain conserved

$$E_{\text{tot}}(\phi_{\text{sr}} \simeq 0.6M_{\text{Pl}}) \simeq 7.3 \cdot 10^{-10} M_{\text{Pl}}^4. \quad (7.12)$$

There is only one brane collision (at $u_1 = 1$, corresponding to $\phi \simeq 0.2M_{\text{Pl}}$) left outside the slow-roll regime.¹⁰ We need to calculate the velocity $\tilde{v}(u_1 = 1)$ from energy conservation, i.e. from

$$\begin{aligned} E_{\text{kin}}(u_1 = 1) &= \frac{1}{2} \dot{\phi}(2)^2 + \mu^{10/3} \left(\phi(2)^{2/3} - \phi(1)^{2/3} \right) \\ &\quad - \frac{g_{\phi\chi}^{5/2}}{(2\pi)^3} \sum_{i=3}^5 v^{3/2}(u_1 = i) [\phi(2) - \phi(1)] \\ &\quad - \frac{g_{\phi\chi}^{5/2}}{(2\pi)^3} v^{3/2}(u_1 = 2) [\phi(2) - \phi(1)]. \end{aligned} \quad (7.13)$$

Note that at $u_1 = 2$ we are just at the end of the slow-roll phase, so we can still obtain $v(u_1 = 2)$ from the slow-roll trajectory. From this equation we find for the kinetic energy at the next-to-last brane crossing

$$E_{\text{kin}}(u_1 = 1) \simeq 8.7 \cdot 10^{-10} M_{\text{Pl}}^4. \quad (7.14)$$

This kinetic energy feeds into the calculation of the string production rate at the $u_1 = 1$ encounter

$$n_{\chi}(u_1 = 1) = \frac{(g_{\phi\chi} \tilde{v}(1))^{3/2}}{(2\pi)^3} = \frac{(g_{\phi\chi} \sqrt{2E_{\text{kin}}})^{3/2}}{(2\pi)^3}. \quad (7.15)$$

Then, using eq. (6.15) we determine the final kinetic energy at $u_1 = 0$, or $\phi_0 = 0$, respectively, to be

$$E_{\text{kin}}(u_1 = 0) \simeq 7.3 \cdot 10^{-10} M_{\text{Pl}}^4. \quad (7.16)$$

Comparing to eq. (7.12), we see that within our rounding accuracy, the entire energy in the system present at ϕ_{sr} has been converted into kinetic energy at $\phi_0 = 0$. That is, the additional potential energy drained by the attached strings is negligibly small. The correction they induce is of the order of 0.2%. Therefore, even after the refined calculation, the high reheat temperature estimates found above persist.

This could give rise to a potential gravitino problem in our model. However, the background constructed in [18] breaks supersymmetry at a very high scale, the lowest KK scale. In the case at hand, this corresponds to

$$m_{\text{KK}} = \frac{2\pi}{\sqrt{\alpha'} L_{u_1}}, \quad (7.17)$$

¹⁰Note that we can still use the normalization and potential eqs. (3.14) and (3.9), respectively, since $u_{1,\text{crit}} \simeq 0.7 < 1$.

as the u_1 direction describes the largest extension of the torus [this follows from eqs. (A.4) and (A.6)]. With the values from the appendix we obtain $m_{\text{KK}} \approx 4 \cdot 10^{-4} M_{\text{Pl}}$, which is about two orders of magnitude smaller than the string scale [see (A.14)]. Since the reheating temperature is slightly larger than the scale of supersymmetry breaking, there is a potential gravitino problem (overabundance of gravitinos produced after reheating [28]). This is not the topic of our work. However, we would like to mention that there are various ways to mitigate the gravitino problem. One way is to invoke a period of thermal inflation at late times [29], another one is to make use of nonperturbative decay channels of the gravitino [30].

8. Conclusions

Recently, it has been proposed to exploit the mechanism of monodromy to achieve a large field range for the inflaton in string-motivated models. Traditionally, the field range had proven to be generically small in these scenarios due to the finite size of the compactified extra dimensions, making a sizeable contribution of tensor perturbations (gravity waves) to the cosmological perturbation spectrum hard to obtain. Monodromy models provide a promising ansatz to overcome this previous phenomenological "no-go theorem" for tensor perturbations from string theory.

We have studied the mechanism of reheating in the model proposed in ref. [4], modelling the Standard Model by a $D6$ brane at a fixed position in the monodromy cycle that the $D4$ brane unwraps while inflation is under way. We have shown that there is virtually no energy transferred when the branes collide during inflation (even though these collisions occur repeatedly), the entire reheating being produced at the last brane encounter. This is reassuring in the sense that the additional $D6$ "stuck in the way" of the inflationary $D4$ does not make it harder to achieve the required number of e-folds. We find that the reheat temperature comes out generically high in these models. Even for different parameter values one would find that the string production rate is very small due to the small velocity along the slow-roll trajectory. Only if the time between the end of inflation and the last brane crossing becomes considerably longer, then there could be any significant string effects, due to two reasons: first, those strings would be produced at a higher rate because of the larger field velocity and second, they would not be diluted anymore.

We have, of course, studied a rather simplified toy model. It would be interesting to refine our approach by studying a more realistic intersecting brane model, in which the $D4$ would cross a considerably higher number of branes. However, given that we found the open string effect to be negligibly small, we do not expect our conclusions to be altered much if the stacks of branes consist of $N = 1, 2$, or 3 $D6$ branes only (these are the numbers needed for the $SU(3) \times SU(2) \times U(1)$ gauge group of the MSSM [19]). If some of these $D6$ overlap with the $D4$ along one internal direction, we would see another interesting effect emerge — the $D4$ could actually dissolve into one of the $D6$ (as they are not mutually BPS in this configuration) and form a bound state with considerably lower energy than the initial setup. In this case one would observe the usual tachyonic string modes stretched between non-BPS branes.

Generically one would expect warping in flux compactifications. This has been neglected so far as well as the backreaction of the branes onto the geometry. For a small number of $D4$ and $D6$ this appears to be a valid approach. However, for a large number of $D6$ (they are more massive than the $D4$) it should be taken into consideration.

In a variant of the model of ref. [4], it has been proposed [31] to use an axion field as the inflaton. This axion originates from the NS or RR two-form, which is integrated over a two-cycle in the internal geometry and therefore appears as a scalar in the four-dimensional theory. The usual shift symmetry of these fields is broken in the presence of branes, which makes them axionic inflaton candidates. Reheating would then proceed via the closed string sector and requires a different description than the simple field theoretic ansatz we used.

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A. Model parameters

In this appendix, we cite important relations between background parameters along with their default values (taken from ref. [4]), which we use whenever numerical estimates are carried out.

A.1 Useful relations

The string length and the 4d Planck mass are related by

$$\frac{1}{\alpha'} = \frac{(2\pi)^7 g_s^2}{L^6} M_{\text{Pl}}^2, \tag{A.1}$$

where the radial modulus L is a measure of the volume of the torus we are considering [whose coordinates are (x, u_1, u_2)]. We have

$$L^3 = L_u^2 L_x, \tag{A.2}$$

and since we have two copies of this torus (with an orbifold projection), the total compact volume is $V = L^6/2$. L_x is the length scale in the x direction [see eq. (2.1)], and L_u is an averaged length scale in the u_1, u_2 directions given by

$$L_u^2 = L_{u_1} L_{u_2}. \tag{A.3}$$

We define the anisotropy parameter β via

$$\beta = \frac{L_{u_2}}{L_{u_1}} = \frac{L_u^2}{L_{u_1}^2} = \frac{L_{u_2}^2}{L_u^2}. \tag{A.4}$$

A.2 Background parameter values

The length scales can be expressed in terms of the background flux quantum numbers M and K as (see [18])

$$L = c_L \cdot K^{1/6}, \tag{A.5}$$

$$L_x = c_{L_x} \cdot M^{-1/2}, \tag{A.6}$$

$$L_u = \frac{c_L^{3/2}}{c_{L_x}^{1/2}} (KM)^{1/4}. \tag{A.7}$$

The coefficients c_L etc. were chosen with numerical values

$$c_L = 1.7, \quad c_{L_x} = 8.6, \quad \left(c_L^{3/2} / c_{L_x}^{1/2} \right) \simeq 0.75. \tag{A.8}$$

Let us now list the values of β and of the fluxes M, K which were used in [4] to obtain an inflationary model with a sufficient number of e-foldings of slow-roll inflation and with a correct normalization of the power spectrum of cosmological perturbations, values which we use in the text for our numerical estimates:

$$\beta \simeq 0.04 \tag{A.9}$$

$$M \simeq 1 \tag{A.10}$$

$$K \simeq 2.2 \cdot 10^6. \tag{A.11}$$

From (A.5)–(A.7) this leads to the scales

$$L \simeq 19.4, \quad L_x \simeq 8.6, \quad L_u \simeq 29.1. \tag{A.12}$$

Note that this implies in particular that $L_{u1} \simeq 145.5$ is the greatest length scale, because β is small.

The string coupling amounts to

$$g_s \simeq 0.1, \tag{A.13}$$

and for the ratio between the string and the Planck scale we find [see eq. (A.1)]

$$\sqrt{\alpha'} M_{\text{Pl}} = \frac{M_{\text{Pl}}}{M_s} \simeq 117. \tag{A.14}$$

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